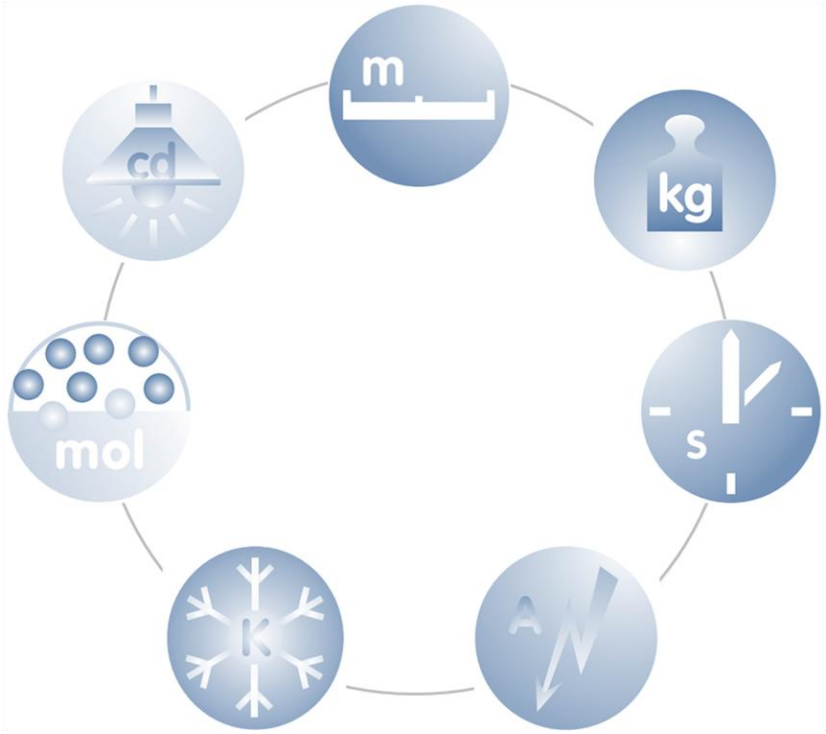
Traceability for computationally-intensive metrology

User manual for

Chebyshev algorithm testing



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Change History

Date	*Revision*	*Rationale*
2015-02-17	1.0	1) Renaming of 'chebychev' to 'chebyshev' 2) Changed element IDs to 'capital letters with underlines' 3) Changed 'certificate' to 'test report' in sections 4.4 (p.9) and 5. 4) Set max points to 631 one p. 6 (former 500) 5) Changes in description of user parameters (p. 7) 6) Changed document version in copyright section 7) Added notification on validity of order key at end of section 3.1 (p. 10) 8) Changed 'certification number' from '007' to '008' (p.2) 9) Added user MPE specification in section 4.3

Please note: the 'Revision' consists of a'major number' – 'dot' – 'minor number' . Work completed regarding major grammar or spelling errors will be marked with a lowercase letter after the revision number.

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1. Introduction

PTBs TraCIM system offers testing of Chebyshev minimization algorithms (Minimum-zone method) for the following basic geometric elements:

Line 2D, Circle 2D, Plane, Sphere and Cylinder

The PTB test was provided within the frame of a project named TraCIM, founded by the European Union (EMRP project NEW06 TraCIM – www.ptb.de/emrp/tracim.html) and is under strict quality control of TraCIM e.V. Association.

The test is provided by the PTB TraCIM Online System. For the test a customer will get an XML file with datasets containing points in space for the elements listed above. The points are both randomly and systematically distributed on fragments of these elements mainly representing full features with different random and systematic form deviation components. A few datasets represent partial features.

Task of the customer is to evaluate the parameters of the best-fit geometric elements regarding the Minimum-Zone-criterion and send these back to the TraCIM System. The TraCIM system automatically compares customer results with its own reference values and sends a report with the test result. Evaluation procedures are subject to section 2.

To order a test it is necessary to provide a client application for communication with the TraCIM Server. Section 3 gives support for the development of a client. The XML data schemes used for data exchange are described in section 4.

Placing an order requires access to the web shop for buying single tests and test packages (web shop: future extension; use e-mail contact until then). After successful purchase of a test a customer will get an order key that allows him to request test data from the TraCIM system with his client application.

In order to check the functionality of his client application, a charge free test suite with public sample data including results is provided. A registered customer is allowed to request a test with these data sets from the TraCIM system at any time in order to check the correct functioning of his client-server communication. See section 3.2 on how to request these test data sets. Additional support will be charged.

1.1. Basic data sets

The Chebyshev elements test contains 50 data sets with following basic data set IDs and associated geometric element IDs:

Table 1: Data/Element IDs

Data set ID	Element ID
C01 – C08	LINE_2D
C09 – C16	CIRCLE_2D
C17 – C26	PLANE
C27 – C36	SPHERE
C37 – C50	CYLINDER

- Each data set consists of points in 3-dimensional Cartesian space.
- A point is represented by the three coordinates (x, y, z).
- For LINE_2D and CIRCLE_2D the z-component of each point is set zero (x, y, 0).
- The amount of points per data set varies between 10 and 631.
- Each point coordinate value is delivered as decimal number with 20 digits and scientific e-format.
- The values refer to the unit mm (millimeter)[1)]
- All point coordinates are within the value range [-500 mm, +500 mm][2)].
- Section 4.2 shows the XML data structure that contains the test data sets.

1) subject to future change
2) preliminary; newer software version may accept customer defined range values.

1.2. Result parameters

For each element the following result parameters must be computed by the test customer:

Table 2: Result parameter list

Element ID	Parameter ID	Description
LINE_2D	Position	The coordinates of the point on the associated line
	Orientation	The direction cosine values of the line
	Form deviation	The form deviation of the line according to Appendix B, ISO 1101:2008
CIRCLE_2D	Position	The coordinates of the center point of the associated circle.
	Radius	The radius of the associated circle
	Form deviation	The form deviation of the circle according to Appendix B, ISO 1101:2008
PLANE	Position	The coordinates of the point on the associated plane
	Orientation	The direction cosine giving the normal vector of the associated plane.
	Form deviation	The form deviation of the plane according to Appendix B, ISO 1101:2008
SPHERE	Position	The coordinates of the center point of the associated sphere.
	Radius	The radius of the associated sphere.
	Form deviation	The form deviation of the sphere according to Appendix B, ISO 1101:2008)
CYLINDER	Position	The coordinates of the point on the associated cylinder axis
	Orientation	The direction cosine values of the associated cylinder axis orientation.
	Radius	The radius of the associated cylinder lateral surface.
	Form deviation	The form deviation of the cylinder according to Appendix B, ISO 1101:2008)

- For all elements the position parameter is a vector in 3-dimensional Cartesian space with components x, y, and z. The values refer to the unit mm (millimeter) - see 1.1 (Basic Data Sets), remark for 'point coordinates.
- For LINE_2D and CIRCLE_2D elements the z-component value of the position parameter must be zero.
- For LINE_2D, PLANE and CYLINDER elements the orientation parameter is a vector in 3-dimensional Cartesian space with components x, y and z. Each vector must be scaled to length value one (direction cosines).
- For LINE_2D elements the z-component of the Orientation parameter must be zero.
- Radius and form deviation must be positive values. These refer to the unit mm (millimeter).
- All parameter values must be in decimal number format (fixed point or floating point e-format are applicable)
- Section 4.3 shows how to write the test results in a file consistent with the TraCIM system test result data structure.

2. Test result evaluation [1)]

During validation the test results of a customer software are compared to the associated reference results in the TraCIM system data base. For this purpose deviations regarding position, orientation, size and form deviation of the geometric elements are evaluated and verified to the specified maximum permissible error values (MPE). The following MPEs are the default values:

Parameter	Specification of deviation	Default MPE value
Position	Orthogonal distance of reference position to test result line, plane, axis or center point	0.1 µm
Orientation	Angle between calculated and reference orientation vector	0.1 µrad
Radius (size)	Difference between calculated radius and reference value	0.1 µm
Form deviation	Difference between calculated form deviation and reference value	0.01 µm

The test passed successfully, if all deviations are less than the preset MPE values. After the evaluation the TraCIM system will issue a test report containing a certificate for tests passed or a table listing the results of the comparison for each of the elements where the test failed. The test report is send back to the customer by the client-server connection.

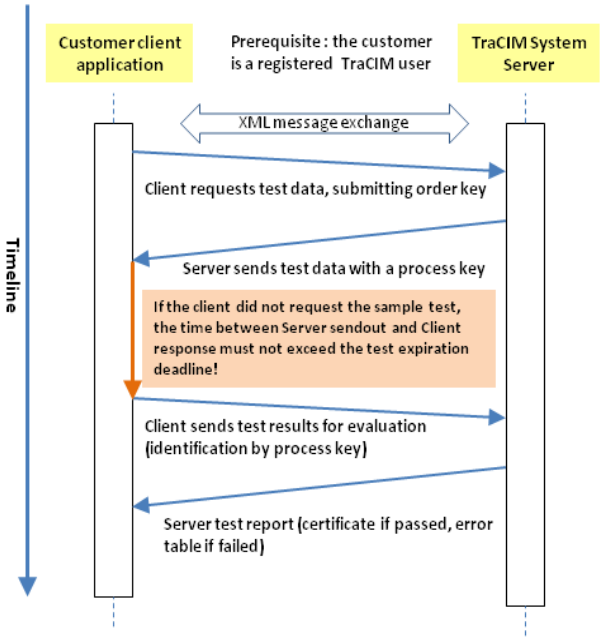
If the customer selects MPEs other than the default values (only larger values are allowed), then the results will be analysed with reference to these values. Giving customer MPEs is shown in section 4.3.

1) 'test expiration' : When a test was requested and the server submitted the data successfully, a 180 day deadline for the customer will start. After this date, test results will not be processed any more. 7 days before the expiration date and time, a notification mail will be sent to the customer's e-mail address (as submitted with the registration data). One day before the expiration date and time, a final notification will be sent.
Additional note: public sample data are available without an expiration date.

3. Client-server communication

The test procedure is highly automated using internet based data exchange by client-server communication. Thru a client application the user is able to obtain test data and send the computational results. The TraCIM system server automatically handles requests. The necessary communication between client and server to perform a single test is shown in the following figure:

Picture 1


Customer client application
Prerequisite : the customer is a registered TraCIM user
TraCIM System Server
XML message exchange
Client requests test data, submitting order key
Server sends test data with a process key
If the client did not request the sample test, the time between Server sendout and Client response must not exceed the test expiration deadline!
Client sends test results for evaluation (identification by process key)
Server test report (certificate if passed, error table if failed)
Timeline

The data exchanged between client and server is encapsulated within XML messages. A proper specification of the applied XML data schemata is subject to section 4. During processing these messages are treated as plain character strings send between the client application and the TraCIM system server.

3.1. Development of a client application

Configuration of the HTTPS connection:

For communication with the TraCIM server a client application must use an HTTPS (Hypertext Transfer Protocol Secure = encrypted HTTPS) connection that allows to send and receive content in the form of character strings containing messages in XML format. Each HTTPS connection is created from a specific URL (Uniform Resource Locator) regarding requests for test data or requests for test result evaluation.
After opening the connection according to the URL the following configurations must be done:

- Enable output and input operations for the connection
- Set the request method “POST” (request comprising input and output)
- Set connection property “content-Type” to “application/xml”
- Set connection property “accept” to “application/xml”
- Set connection property “content-length” to the amount of characters of the content that is send to the TraCIM system.

Packages for creation and configuration of an HTTPS connection are available for different programming languages such as for (examples):

- Java: java.net API (HttpURLConnection)
- C/C++: Microsoft C++ REST SDK (“Casablanca”) or similar
- C#: System.Net (.NET 4.5 : System.Net.Http) Assembly

POST request for obtaining test data sets:

The URL of an HTTPS connection for POST request in order to obtain Chebyshev test data sets is

https://tracim.ptb.de//tracim/api/order/<CHEBYSHEV_ORDER_KEY>/test

where <CHEBYSHEV_ORDER_KEY> has to be replaced by the Chebyshev test order key purchased at the TraCIM web shop (future extension). After creation and configuration of a connection as described above the client application has to send the request message. When the TraCIM system receives the client message it will start creating test data sets and a unique process key. The TraCIM content is returned to the client.

For cases that a customer is not properly registered for the TraCIM System or incorrect order keys the TraCIM server will send an error message.

Client content and TraCIM system content for retrieving test data are presented in sections 4.1 and 4.2. Error messages are presented in section 4.5.

POST request for sending test results and obtaining the certificate:

The URL of an HTTPS connection for a POST request in order to send test results for evaluation and get the certificate for Chebyshev testing is

https://www.tracim.ptb.de/tracim/api/test/<PROCESS_KEY>

where <PROCESS_KEY> has to be replaced by the individual process key that was returned by the TraCIM system at the POST request for test data sets.

The client will prepare the XML content (test result parameters, process key, software identification, ….) to be attached to the POST request and will send it to the TraCIM system.

The TraCIM System will receive and evaluate the content and generate a test report. It states whether the test is passed (user results sufficiently accurate) or gives a table with an error outline for all elements ‘C01-C50’ according to each parameter. In case of a sample test request the test report is not countersigned by PTB seal. The data sent to the client contains the test report encoded in XML.

For the case of an incorrect process key the TraCIM content will be an error message.

Client content and TraCIM system content for retrieving the test evaluation and PDF report are presented in sections 4.3 and 4.4. Error messages are presented in section 4.5.

!	TraCIM service for testing is limited in time. After receiving an order key a customer has a total of 200 days for performing the Chebyshev test. After 20, 120 and 180 days the TraCIM system will automatically send warning messages to the customer e-mail address stating the remaining time for testing. When the validity of a test expires a final information message is send to the customer.

3.2. Public sample data for client testing

Sample data denotes a complete Chebyshev test for backtracking errors within the client application that could put a successful commercial test run on the risk. For any registered customer the data is free of charge with unlimited request amount. In comparison to a test with commercial test data the certificate returned by the server is not countersigned by PTB as legally valid certificate.

The sample order key is available for registered customers. Please check the website (www.tracim.ptb.de) or send a mail to tracim.support@ptb.de for further information.

4. XML message content

4.1. Test request

The XML scheme below shows the complete message for the initial test data request by the client application. The client has to specify the fields [TEST_CLIENT] with its proper name and its version in the field [TEST_CLIENT_VERSION]. Both entries must be UTF-8 formatted character Strings with at least one and a maximum of 20 characters.

```
<?xml version="1.0" encoding="UTF-8" standalone="true"?>
<tracim:tracim
xmlns:xsi=”http://www.w3.org/2001/XMLSchema-instance”
xmlns:chebyshev="http://tracim.ptb.de/chebyshev/test"
xmlns:tracim="http://tracim.ptb.de/tracim">

        <product>[TEST_CLIENT]</product>
        <version>[TEST_CLIENT_VERSION]</version>

</tracim:tracim>
```

4.2. Test data

Test data returned by the TraCIM system composed according to an XML scheme with three major elements for order identification, process identification and the test data sets. The order element contains a copy of the order key, a date of the creation of the test data and a date for the expiration of the test data (deadline for sending test results for evaluation). The process element contains the process key associated with the test data request. The test element contains several point clouds with vector elements that give the x, y and z coordinates of the test data. For each point cloud the field “basicID”

contains the unique data set id (C01-C50) and the field “computationalObject” contains the corresponding geometrical element (LINE_2D, CIRCLE_2D, PLANE, SPHERE, CCYLINDER).

```
<?xml version="1.0" encoding="UTF-8" standalone="true"?>
<tracim:tracim
              xmlns:xsi=”http://www.w3.org/2001/XMLSchema-instance”
              xmlns:chebyshev="http://tracim.ptb.de/chebyshev/test"
              xmlns:tracim="http://tracim.ptb.de/tracim">

          <tracim:order>
            <tracim:key>[ORDER_KEY]</tracim:key>
            <tracim:creationDate>[…]</tracim:creationDate>
            <tracim:expiration Date>[…]</tracim:expirationData>
          </tracim:order>

          <tracim:process>
            <tracim:key>[PROCESS_KEY]</tracim:key>
          </tracim:process>

          <tracim: tests xsi:type=”chebyshev:chebyshevTestPackage”>
            <chebyshev:package>
              <chebyshev:testElement>
                <chebyshev:basicID>C01</chebyshev:basicID>
                <chebyshev:computationObject>LINE_2D</chebyshev:computationObject>
                <chebyshev:pointCloud>

                  <chebyshev:vectors>
                    <chebyshev:x>-44.7893023434000000</chebyshev:x>
                    <chebyshev:y>58.34322222322222200</chebyshev:y>
                    <chebyshev:z>0.000000000000000000</chebyshev:z>
                  </chebyshev:vectors>

                  <chebyshev:vectors>
                    <chebyshev:x>-44.7893023434000000</chebyshev:x>
                    <chebyshev:y>58.34322222322222200</chebyshev:y>
                    <chebyshev:z>0.000000000000000000</chebyshev:z>
                  </chebyshev:vectors>
                    […]
                  </chebyshev:pointCloud>
                </chebyshev:testElement>

                <chebyshev:testElement>
                  <chebyshev:basicID>C37</chebyshev:basicID>
                  <chebyshev:computationObject>CYLINDER</chebyshev:computationObject>
                    […]
                </chebyshev:testElement>
                  […]
              </chebyshev:package>
            </tracim:tests>
          </tracim:tracim>
```

4.3.Test results

For sending the calculated results of software under test to the TraCIM system the XML scheme below must be used. The client has to fill in the following information (next page):

- [PROCESS_KEY] key received in message from section 4.2
- [CUSTOMER_NAME] customer name from TraCIM system registration
- [SOFTWARE_NAME] name of software that computed the test results
- [SOFTWARE_VERSION] version of software that computed the test results
- [SOFTWARE_REVISION] revision of software that computed the test results

Further the client has to specify the MPE values for position, orientation, size and form deviation that should be used for test evaluation. User MPE values are subjected to unit millimeter for length and rad for orientation.

This information is followed by elements that contain the result parameters of the software under test. For each test data set the client has to specify a single results element set containing the basic data set id (C01 – C50) and the geometrical element id (LINE_2D, CIRCLE_2D, PLANE, SPHERE, CYLINDER).

```
<?xml version="1.0" encoding="UTF-8" standalone="true"?>
<chebyshev:chebyshevResultPackage
       xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
       xmlns:chebyshev="http://tracim.ptb.de/chebyshev/test"
       xmlns:tracim="http://tracim.ptb.de/tracim">

  <chebyshev:processKey>[PROCESS_KEY]</chebyshev:processKey>
  <chebyshev:customer>[CUSTOMER_NAME]</chebyshev:customer>
  <chebyshev:softwareName>[SOFTWARE_NAME]</chebyshev:softwareName>
  <chebyshev:softwareVersion>[SOFTWARE_VERSION]</chebyshev:softwareVersion>
  <chebyshev:softwareRev>[SOFTWARE_REVISION]</chebyshev:softwareRev>

  <chebyshev:mpe_position>0.0001</chebyshev:mpe_position>
  <chebyshev:mpe_orientation>0.0000001</chebyshev:mpe_orientation>
  <chebyshev:mpe_size>0.0001</chebyshev:mpe_size>
  <chebyshev:mpe_formDeviation>0.00001</chebyshev:mpe_formDeviation>

  <chebyshev:resultPackage>
    <chebyshev:results>
      <chebyshev:basicID>C01</chebyshev:basicID>
      <chebyshev:computationObject>LINE_2D</chebyshev:computationObject>
      <chebyshev:refParameter>
        <chebyshev:positionX>0.000001</chebyshev:positionX>
        <chebyshev:positionY>0.000005</chebyshev:positionY>
        <chebyshev:positionZ>0.000000</chebyshev:positionZ>
        <chebyshev:orientationX>1.00000000021</chebyshev:orientationX>
        <chebyshev:orientationY>-0.00000000005</chebyshev:orientationY>
        <chebyshev:orientationZ>0.0000000000</chebyshev:orientationZ>
        <chebyshev:formDeviation>0.0003578</chebyshev:formDeviation>
      </chebyshev:refParameter>
    </chebyshev:results>

    <chebyshev:results>
      <chebyshev:basicID>C37</chebyshev:basicID>
      <chebyshev:computationObject>CYLINDER</chebyshev:computationObject>
      […]
    </chebyshev:results>
      […]
  </chebyshev:resultPackage>

</chebyshev:chebyshevResultPackage>
```

According to the result parameter list in section 1.2. a client has to provide the following types of parameter elements:

- positionX x-coordinate of the position parameter
- positionY y- coordinate of the position parameter
- positionZ z- coordinate of the position parameter
- orientationX x- component of the orientation vector
- orientationY y- component of the orientation vector
- orientationZ z- component of the orientation vector
- radius radius for ‘CIRCLE_2D’, ‘SPHERE’ and ‘CYLINDER’
- formDeviation form deviation associated with the Chebyshev element

Entries for parameters that are not part of the geometrical element parameter specification have not to be written in the scheme (in the example above this applies for the radius parameter in the context of LINE_2D result parameters). Exceptions are the z-coordinate and z-component of position and orientation for the two-dimensional elements LINE_2D and CIRCLE_2D. The client must fill in these with value zero. All values that are entered into the scheme must apply with the requirements in section 1.2. In addition these values have to be character strings compatible with UTF-8 format.

4.4. Test report

After evaluation the TraCIM system returns XML data according to a XML scheme with the validation result. It comprises the three elements

- Passed true, if the software passed the test successful, else false
- Report character String with short report on the test evaluation
- reportPDF character String with the test report PDF

In order to create the test report PDF document as PDF file, the “reportPDF” character string can be converted and written to new file with the proper “.pdf” file name extension.

```
<?xml version="1.0" encoding="UTF-8" standalone="true"?>
<tracim:tracim
xmlns:xsi=”http://www.w3.org/2001/XMLSchema-instance”
xmlns:chebyshev="http://tracim.ptb.de/chebyshev/test"
xmlns:tracim="http://tracim.ptb.de/tracim">

        <tracim:validation>
          <tracim:passed>true</tracim:passed>
          <tracim:report></tracim:report>
          <tracim:reportPDF>1)JKD5iuDUD098IHh[…]</tracim:reportPDF>
        </tracim:validation>

</tracim:tracim>
```

1) ‘base64’ encoded

4.5. TraCIM Error Messages

The following XML code is sent by the server in case of errors, where [error code] and [error description] are replaced with values according to table 4 (after the XML excerpt).

```
<?xml version="1.0" encoding="UTF-8" standalone="true"?>
<tracim:tracim
       xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
       xmlns:chebyshev="http://tracim.ptb.de/chebyshev/test"
       xmlns:tracim="http://tracim.ptb.de/tracim">
  <tracim:error>
    <tracim:code>[error code]</tracim:code>
    <tracim:description>[error description]</tracim:description>
  </tracim:error>
</tracim:tracim>
```

Table 4 : Error messages [1)]

[error code]	[error description]
403	No more tests left for this test process.
403	Test process already closed.
403	Test Data already used.
404	Unknown order number.
422 [2)]	Unknown request type. Consult specific message.
422 [2)]	Process key does not match!
422 [2)]	Customer element is blank!
422 [2)]	Software vendor element is blank!
422 [2)]	Software name element is blank!
422 [2)]	Software version element is blank!
422 [2)]	MPE-elements not present. You are allowed to set MPE's to zero but the elements have to be present!
422 [2)]	Check your MPEs! If you set one MPE you have to set it all!
422 [2)]	Check your MPEs! Negative MPE's are not allowed.
500	Internal Server error.

Notes to table 4

[1)] Column [error code]: these codes are identical with the HTTP error codes. As one code can describe more than one reason, multiple lines with the same code are present; details can be found in the next column ([error description]).

Column [error description]: a textual explanation of the error reason.

[2)] For this code, a 'frame message' will be generated – '*Sorry, but your request could not be interpreted. Please check your message or contact the support. The system says:* '[error description]

Please note: if, due to specific reasons, the server can not generate a proper XML error message, please check the HTML response of the server. This can be done i.e. with programming the client for processing both XML and HTML messages.

4.6 TraCIM schemata

The XML schemata can be obtained from the following URL's: *(preliminary!)*

https://tracim.ptb.de/tracim/schema/PTBWHZ_MATH_CHEBYSHEV_v1_test.xsd[1)]
https://tracim.ptb.de/tracim/schema/PTBWHZ_MATH_CHEBYSHEV_v1_result.xsd [2)]
https://tracim.ptb.de/tracim/schema/tracim.xsd[3)]

Notes:

1) Test data schema; 2) Result data schema; 3) Report schema

It is strongly recommended, that a customer in order to test client functions uses the schemata during client development.

5. The test report

The report can be extracted from the string stream (4.4, XML tag ‘tracim: reportPDF’ – ‘base64’ encoded) into a PDF file ready for printing. The layout follows the rules for test reports as used in the PTB (official header, legal statements etc.)

6. How to get support

- technical support for client software : tracim.support@ptb.de
- support regarding computational aims : tracim.support@ptb.de
- Provider support (contact, fees, …) : tracim.support@ptb.de

7. Legal and copyright information

Copyright for this document: PTB. The document content is subject to change without notice.
Version note: V1.0 as of February 2014.

8. Appendices

Appendix A – Reference parameter values for example test data sets

Version 2014-10-24

Tables ‘a’ to ‘e’ are showing reference parameter values for the public test data sets of elements Line 2D, Circle 2D, Plane, Sphere and Cylinder. These are decimal numbers up to 10 digits precision. Values for position, radius and form deviation are subjected to the dimension millimeter. The components of Orientation are values in length units (no physical dimension).

a) Line 2D

Basic ID	Position	Orientation	Form deviation	Number of points
C01	x: -10.0 y: -100.0	x: 1.0 y: 0.0	0.022	260
C02	x: -240.0 y: 50.0	x: 0.0 y: 1.0	0.02	631
C03	x: 410.0 y: 405.0	x: 0.7071067811865 y: 0.7071067811865	0.001	120
C04	x: 130.0 y: 5.0	x: -0.7071067811865 y: 0.7071067811865	0.005	50
C05	x: -95.0 y: -103.0	x: 0.8944271909999 y: 0.4472135954999	0.002	10
C06	x: 5.0 y: -340.0	x: 0.4472135954999 y: 0.8944271909999	0.015	161
C07	x: 300.0 y: -250.0	x: -0.0995037190209 y: 0.9950371902099	0.05	80
C08	x: -125.0 y: 345.0	x: -0.9977851578566 y: 0.0665190105237	0.01	331

b) Circle 2D

Basic ID	Position	Radius	Form deviation	Number of points
C09	x: 400.0 y: 390.0	20.0	0.05	250
C10	x: -200.0 y: 180.0	200.0	0.022	120
C11	x: -250.0 y: -250.0	125.0	0.005	260
C12	x: 105.0 y: -57.0	250.0	0.02	250
C13	x: 0.0 y: -305.0	50.0	0.005	78
C14	x: 280.0 y: -300.0	12.0	0.002	216
C15	x: 100.0 y: 120.0	25.0	0.001	30
C16	x: 150.0 y: 50.0	65.0	0.01	450

c) Plane

Basic ID	Position	Orientation	Form deviation	Number of points
C17	x: 0.0 y: 0.0 z: 0.0	x: 0.0 y: 1.0 z: 0.0	0.075	50
C18	x: 400.0 y: 400.0 z: 400.0	x: 1.0 y: 0.0 z: 0.0	0.001	145
C19	x: 350.0 y: 0.0 z: -250.0	x: 0.0 y: 0.0 z: 1.0	0.01	397
C20	x: 0.0 y: -270.0 z: -280.0	x: 0.0 y: 0.7071067811865 z: 0.7071067811865	0.005	136
C21	x: 310.0 y: -95.0 z: 210.0	x: 0.7071067811865 y: 0.0 z: 0.7071067811865	0.003	10
C22	x: 185.0 y: 155.0 z: -10.0	x: 0.7071067811865 y: 0.7071067811865 z: 0.0	0.005	20
C23	x: 110.0 y: 10.0 z: -310.0	x: 0.5934424260562 y: 0.6593804733957 z: 0.46156633137705	0.022	174
C24	x: 20.0 y: -15.0 z: 20.0	x: -0.6388765649999 y: 0.5111012519999 z: 0.5749889084999	0.05	450
C25	x: -120.0 y: 130.0 z: 170.0	x: 0.5232045649263 y: -0.5232045649263 z: 0.6726915834767	0.09	451
C26	x: -50.0 y: -60.0 z: -50.0	x: -0.5533715710928 y: -0.6225430174794 z: 0.5533715710928	0.025	35

d) Sphere

Basic ID	Position	Radius	Form deviation	Number of points
C27	x: -280.0 y: -300.0 z: -220.0	70.0	0.015	144
C28	x: -250.0 y: 240.0 z: 250.0	130.0	0.02	350
C29	x: 150.0 y: -120.0 z: -90.0	25.0	0.005	25
C30	x: 150.0 y: 150.0 z: -110.0	210.0	0.04	182
C31	x: 100.0 y: 350.0 z: 0.0	15.0	0.003	50
C32	x: 300.0 y: 0.0 z: 280.0	20.0	0.02	241
C33	x: -90.0 y: -90.0 z: 0.0	55.0	0.01	55
C34	x: 0.0 y: 0.0 z: 0.0	5.0	0.001	120
C35	x: 0.0 y: 0.0 z: 0.0	260.0	0.05	120
C36	x: 280.0 y: -200.0 z: 300.0	60.0	0.01	210

e) Cylinder

Basic ID	Position	Orientation	Radius	Form deviation	Number of points
C37	x: 0.0 y: 0.0 z: 0.0	x: 1.0 y: 0.0 z: 0.0	250.0	0.025	250
C38	x: 230.0 y: 310.0 z: 260.0	x: 0.0 y: 1.0 z: 0.0	50.0	0.005	75
C39	x: -250.0 y: -250.0 z: -250.0	x: 0.0 y: 0.0 z: 1.0	12.5	0.002	216
C40	x: 90.0 y: -110.0 z: -80.0	x: 0.7071067811865 y: 0.7071067811865 z: 0.0	20.0	0.01	250
C41	x: -90.0 y: -200.0 z: 100.0	x: 0.0 y: 0.7071067811865 z: 0.7071067811865	200.0	0.02	112
C42	x: 240.0 y: 280.0 z: -235.0	x: 0.7071067811865 y: 0.0 z: 0.7071067811865	37.5	0.005	264
C43	x: 60.0 y: 50.0 z: 80.0	x: 0.8479983040051 y: 0.3179993640019 z: 0.4239991520025	125.0	0.05	480
C44	x: 0.0 y: 0.0 z: 0.0	x: -0.5391638660171 y: 0.5391638660171 z: 0.6469966392206	92.0	0.015	125
C45	x: -20.0 y: -20.0 z: 20.0	x: -0.3810003810005 y: -0.8890008890013 z: 0.2540002540003	65.0	0.015	455
C46	x: 300.0 y: -415.0 z: 320.0	x: 0.2407717061715 y: -0.4815434123431 z: 0.8427009716003	25.0	0.001	30

Next page: Cylinder values continued

Cylinder values continued from previous page

Basic ID	Position	Orientation	Radius	Form deviation	Number of points
C47	x: 55.0 y: 50.0 z: 50.0	x: 0.1230914909793 y: 0.9847319278346 z: 0.1230914909793	3.25	0.005	40
C48	x: 50.0 y: 45.0 z: -45.0	x: 0.9630868246861 y: -0.1203858530857 z: 0.2407717061715	4.35	0.008	66
C49	x: -55.0 y: 50.0 z: 50.0	x: -0.1162476387438 y: 0.9299811099505 z: 0.3487429162314	5.375	0.005	50
C50	x: 50.0 y: -50.0 z: 50.0	x: -0.9630868246861 y: -0.1203858530857 z: 0.2407717061715	1.65	0.002	30

Appendix B – Underlying computational aims

Version 2014-10-24

The input data for the Minimum-Zone element fitting is a so called point cloud

$$P \coloneqq \{P_1, \dots, P_n\}$$

of $n \in \mathbb{N}$ point coordinates $P_i \in \mathbb{R}^k$, $k = 2,3$ for all indices i from the index set

$$I \coloneqq \{1, \dots, n\}.$$

The point cloud represents a discrete sample of surficial measurement points from a workpiece that has the shape of a basic geometric element lateral surface in the 3D-case or respectively a lateral curvature in the 2D-case. The elements subject to this appendix are Line 2D, Circle 2D, Plane, Sphere and Cylinder. By

$$b \in G \subset \mathbb{R}^m$$

with $m \in \mathbb{N}, m \geq 1$ and $G \neq \emptyset$ we introduce a parameter vector that defines the upper elements in their ideal geometrical shape. The dimension m and range of valid parameters G have individual definitions for each geometric element. Then for a fixed parameter b each of the ideal shaped geometric elements is implicitly defined by

$$A(b) \coloneqq \{v \in \mathbb{R}^k | f(v, b) = 0\}$$

where $f: \mathbb{R}^k \times \mathbb{R}^m \to \mathbb{R}$ denotes a function that gives the orthogonal distance between an arbitrary point in $\mathbb{R}^k$ and the ideal surface or curvature of the geometric element.

In general there may not be a parameter vector b that satisfies $P \subseteq A(b)$ due to deviations in form and size of the workpiece in comparison with the underlying ideal geometric element. Hence applications in geometrical dimensioning and tolerancing require additional fitting criteria to provide definitions of associated geometrical elements for the point clouds.

One of these criteria is the Minimum-Zone best-fit. Its objective is

$$\min_{b \in G} \max_{i \in I} |f_i(b)| \tag{1}$$

and

$$f_i(b) \coloneqq f(P_i, b)$$

give the orthogonal distance of the points from the point cloud P to the geometric element with parameters b. Problem (1) is referred to as a discreet minimax-problem in optimization theory. Moreover the objective

$$F(b) \coloneqq \max_{i \in I} |f_i(b)|$$

is the norm of all orthogonal distance values in the maximum norm on the real value vector space $\mathbb{R}^n$ that is commonly known as Chebyshev-norm and hence (1) is called Chebyshev-fitting, too. The table below gives formal mathematical definitions for parameter vectors, parameter space and orthogonal distance functions.

Table 1: *Geometry parameter and distance function definitions for Minimum-Zone elements Line 2D, Circle 2D, Plane, Sphere and Cylinder.*

Element	**Parameter vectors b**	**Range of parameters G**	**Distance function**
Circle 2D	$b \coloneqq (c_x, c_y, r)^T$	$\mathbb{R}^2 \times \mathbb{R}_+$	$f(Q,b) \coloneqq \|\|Q - C\|\| - r$
	Center $C = (c_x, c_y)^T$	$\mathbb{R}^2$	
	Radius r	$\mathbb{R}_+$	
Line 2D	$b \coloneqq (c_x, c_y, v_x, v_y)^T$	$\mathbb{R}^2 \times S_1(\mathbf{0})$	$f(Q,b) \coloneqq \langle Q - C, n \rangle$ with $n \coloneqq (-v_y, v_x)^T$
	Orthogonal projection of point data set barycenter on line $C = (c_x, c_y)^T$	$\mathbb{R}^2$	
	Line direction $v = (v_x, v_y)^T$	$S_1(\mathbf{0}) \subset \mathbb{R}^2$	
Plane	$b \coloneqq (c_x, c_y, c_z, n_x, n_y, n_z)^T$	$\mathbb{R}^3 \times S_1(\mathbf{0})$	$f(Q,b) \coloneqq \langle Q - C, n \rangle$
	Orthogonal projection of point data set barycenter on plane $C = (c_x, c_y, c_z)^T$	$\mathbb{R}^3$	
	Plane normal $n = (n_x, n_y, n_z)^T$	$S_1(\mathbf{0}) \subset \mathbb{R}^3$	
Sphere	$b \coloneqq (c_x, c_y, c_z, r)^T$	$\mathbb{R}^3 \times \mathbb{R}_+$	$f(Q,b) \coloneqq \|\|Q - C\|\| - r$
	Center point $C = (c_x, c_y, c_z)^T$	$\mathbb{R}^3$	
	Radius r	$\mathbb{R}_+$	
Cylinder	$b \coloneqq (c_x, c_y, c_z, v_x, v_y, v_z, r)^T$	$\mathbb{R}^3 \times S_1(\mathbf{0}) \times \mathbb{R}_+$	$f(Q,b) \coloneqq \|\|(Q - C) \times v\|\| - r$
	Orthogonal projection of point data set barycenter on cylinder axis $C = (c_x, c_y, c_z)^T$	$\mathbb{R}^3$	
	Axis direction $v = (v_x, v_y, v_z)^T$	$S_1(\mathbf{0}) \subset \mathbb{R}^3$	
	Radius r	$\mathbb{R}_+$	

Computation of projected centroids:

The centroid of the point cloud (point data set barycenter) is

$$\bar{P} \coloneqq \frac{1}{n} \sum_{i=1}^{n} P_i.$$

Then table 2 gives the evaluation of the projected centroid for several elements.

Element	**Computation projected centroid C**
Line 2D, Cylinder	C solves $\langle \bar{P} - C, v \rangle = 0$
Plane	C solves $(\bar{P} - C) \times n = 0$

Table 2: *Rules for projected centroid computation*

Computation of element form deviation:

For each geometrical element the form deviation is defined by

$$ef := 2F(b).$$

For parameters b from the Minimum-Zone best-fit (1) this yields form deviation assessment according to requirements from ISO 1101 – Appendix B [4]. The form deviation describes the width of an area enclosing all points in the point cloud – the minimum-zone. The following geometrical depictions apply for Chebyshev elements:

- **Line 2D:** The Minimum-Zone is bounded by two geometrical ideal straight lines with minimum distance that are parallel to the Chebyshev line. The orthogonal distance of the bounding lines is the associated form deviation. The Chebyshev line is the median line between the bounding lines.

- **Circle 2D:** The Minimum-Zone is bounded by two geometrical ideal concentric circles with center point coincident with the center point of the Chebyshev circle and minimum radius distance. The radius distance is the associated form deviation. The Chebyshev circle is the median circle between the bounding circles.

- **Plane:** The Minimum-Zone is bounded by two geometrical ideal planes with minimum distance that are parallel to the Chebyshev plane. The orthogonal distance of the bounding planes is the form deviation. The Chebyshev plane is the median plane between the bounding planes.

- **Sphere:** The Minimum-Zone is bounded by two geometrical ideal concentric spheres with center point coincident with the center point of the Chebyshev sphere and minimum radius distance. The radius distance is the associated form deviation. The Chebyshev sphere is the median sphere between the bounding spheres.
- **Cylinder:** The Minimum-Zone is bounded by two geometrical ideal cylinders coaxial to the Chebyshev cylinder and with minimum radius distance. The form deviation is the radius distance is the associated form deviation. The Chebyshev cylinder is the median cylinder between the bounding cylinders.

Remarks on properties of the Chebyshev element fitting:

a) The parameter selection in Table 1 is subjected to normative specifications for geometrical element algorithm testing in ISO 10360 part 6 [1]. For computational evaluation of the elements with metrological software different parameterization is possible.

b) Depending on the structure of the point cloud problem (1) could have a unique global solution, several equivalent global solutions or even no solution. For example symmetric data sets may contain equivalent global solutions. Moreover for collinear points the fitting of Circle 2D has an invalid solution with finite radius.

c) Despite the global solution properties a point cloud could contain additional local solution that are consistent with mathematical solution properties for problem (1) but have a greater value for form deviation than a global solution. Hence the application of numerical methods for the element computation requires the application of additional global search strategies in order to find a global solution instead of a local solution.

d) For solving problem (1) it is recommended to use numerical methods that are able to

Further references for details:

[1] DIN EN ISO 10360-6: *Geometrical Product Specifications (GPS) – Acceptance and reverification tests for coordinate measuring machines (CMM) – Part 6: Estimation of errors in computing Gaussian associated features (ISO 10360-6:2001+Cor. 1:2007)*, German version EN ISO 10360-6:2001+AC:2007

[2] G.T. Anthony, B. Bittner, R. Drieschner, J. Kok, et. al.: *Chebyshev reference software for the evaluation of coordinate measuring machine data*, Commission of the European Communities, Programme for Applied Metrology and Chemical Analysis (BCR), Report EUR 15304 EN, October 1993

[3] R. Drieschner: *Chebyshev approximation to data by geometric elements*, Numerical Algorithms 5, pp 509-522, 1993

[4] DIN EN ISO 1101: *Geometrical Product Specification (GPS) – Geometrical tolerancing – Tolerances of form, orientation, location and run-out (ISO 1101:2004);* German version EN ISO 1101:2005; August 2008